Markovian Logics: The Metric Space of Logical Formulas

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A central questions in the field of probabilistic and Markovian systems is *"when do two systems behave similarly up to some observational error?"*. *Probabilistic and stochastic bisimulations* relates models with identical behaviours and are characterized by multimodal logics with operators indexed with transition probabilities or rates [3, 4, 5]. In spite of their elegant theories, these concepts are too strict for applications where the parameters of a model are approximated. Thus, the interest shifts from identical to similar behaviours and the bisimulations are replaced by a class of pseudometrics [5]: two processes are at distance zero iff they are bisimilar and are close when they differ by a small amount in their quantitative behaviours. These pseudometrics are defined on top of the logics that characterize bisimulations: the satisfiability relation $P \Vdash \phi$ (for a process $P \in \mathbb{P}$ and a logical formula $\phi \in \mathcal{L}$) is approximated by a function $d : \mathbb{P} \times \mathcal{L} \to [0, 1]$ such that $d(P, \phi) = 0$ iff $P \Vdash \phi$. d induces a distance D on processes by $D(P, P') = sup\{|d(P, \phi) - d(P', \phi)|, \phi \in \mathcal{L}\}$ and similarly, a pseudometric δ on formulas by $\delta(\phi, \psi) = sup\{|d(P, \phi) - d(P, \psi)|, P \in \mathbb{P}$. In the context of complete axiomatizations for the probabilistic and stochastic logics [1, 2, 3, 6], one can prove that δ is a measure of provability: $\delta(\phi, \psi)$ measures the amount of hypothesis that can prove one while disproving the other and consequently, $\delta(\phi, \psi) = 0$ iff $\vdash \phi \leftrightarrow \psi$.

Because $\mathcal{L}$ and $\mathbb{P}$ are infinite, D and δ are not always computable. In some cases, already evaluating $d(P, \phi)$ is problematic. However, relying on the fact that these logics enjoy the finite model property [3, 6], we can define algorithmically an approximation δ_ε of δ, parametrized by an error $\varepsilon > 0$: for any process P and formulas ϕ, ψ, $d(P, \phi) \leq d(P, \psi) + \delta_\varepsilon(\phi, \psi) + \varepsilon$. This is a robustness result useful in applications where one needs to decide what is the probability for a system to satisfy a certain property, given the probability that it satisfies an other property.

References

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